The Two-fluid Description of a Mesoscopic Cylinder

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Abstract

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1 Introduction

The transport properties of mesoscopic metallic or semiconducting samples at low temperature have been shown to exhibit features characteristic of the quantum coherence of the electronic wave function along the whole sample, see e.g. [1]. It is thus interesting to compare the characteristics of mesoscopic systems with those for other coherent systems.

The aim of the present work is to show that if we reduce the dimensions of a cylinder made of a normal metal or a semiconductor to mesoscopic dimensions and if we assume that electrons interact via the magnetostatic interaction, a system exhibits coherent properties absent in macroscopic samples.

Coherent systems like superfluids or superconductors can be in general described by a two-fluid model namely the total density n is equal to the sum of the densities of the coherent n_c and normal n_n components

$$n = n_c + n_n. \tag{1}$$

For macroscopic, nonsuperconducting systems (here after refered as "normal") $n_c = 0$.

In this paper we discuss the electromagnetic response and its static limit of a mesoscopic cylinder of very small thickness d made of a normal metal or semiconductor. The circumference and height of the cylinder are denoted by L_x, L_y respectively ($d \ll L_x, L_y$).

We argue that such mesoscopic cylinder can be in general described by the two-fluid model. The coherent state encountered here, although in principle different from the superconducting state, exhibits the following properties which bear resemblance to superconductors:

- free acceleration of part of electrons
- strong diamagnetic susceptibility at low fields due to the reduction of paramagnetic susceptibility
- dynamic energy gap coming from magnetostatic coupling of electrons
- persistent currents in the presence of the static flux

- flux trapping

The relation between the induced current density $J(\underline{q}, w)$ and the electric field $E(\underline{q}, w)$ is generally written as

$$J(\underline{q}, w) = \frac{iK(\underline{q}, w)}{w} E(\underline{q}, w). \tag{2}$$

It is the form of the kernel $K(\underline{q}, w)$ that determines the properties of the system both static as well as for finite frequencies. The following limits imply

$$\lim_{w\to 0}\lim_{\underline{q}\to 0} K(\underline{q}, w) \equiv -\chi \neq 0 \quad \text{- infinite conductivity,} \tag{3}$$

$$\lim_{\underline{q}\to 0}\lim_{w\to 0} K(\underline{q}, w) \neq 0 \qquad \text{- Meissner effect .} \tag{4}$$

For normal electrons in macroscopic metallic or semiconducting samples both limits are zero.

In this paper we calculate the conductivity $\sigma(w)$ and we study the first limit only as we are dealing with a narrow cylinder and we neglect the q dependence of the vector potential $\mathbf{A}$. The results for a thick cylinder will be presented in a forthcoming paper.

We show that the relation (3) is fulfilled in the nonsuperconducting mesoscopic cylinder and its magnitude depends on the shape of the Fermi Surface (FS).

The reaction of a one-dimensional mesoscopic ring to a static and time varying magnetic field and coupled to a thermal bath has been investigated by Trivedi et. al. [2]. They discussed using the Kubo linear response theory the properties of the dynamic and static response function. Such a one channel system is a simplification - experimental samples have many transverse channels and the question arises how do the system characteristics depend on the transverse dimensions.

We extend the calculations of the conductivity of a one-dimensional disordered ring [2] driven by the electromagnetic field to the system of cylinder geometry and check whether coherence will still be maintained in a system with many transverse channels.

We also discuss the orbital magnetic susceptibility and show that it exhibits anomalous diamagnetism related to the presence of a dynamic gap.

We further investigate persistent currents which arise as a result of the flux sensitivity of energy levels. We discuss coherence of currents from different channels and the magnitude of persistent currents for different shapes of the FS.

In one of our recent papers [3] we discussed persistent, paramagnetic currents in the free electron model with different shapes of the FS. The FS was assumed to be a function of certain parameters which described a change of its shape from circular to nearly quadratic. In this paper we use the tight-binding model where different shapes of the FS follow in a natural way from the crystal structure and band filling [4]. We discuss here, contrary to [3], diamagnetic persistent currents.

Finally we address the issue of whether a system of interacting electrons confined to a mesoscopic cylinder can sustain a spontaneous persistent currents in the absence of an externally applied flux. These currents correspond to flux trapping in the cylinder.

2 Coherent properties of a mesoscopic cylinder

Let us consider a system of spinless electrons constrained to move on a hollow cylinder with M channels in the height L_y and N sites with lattice spacing a in the circumference L_x threated by the magnetic flux ϕ. We assume that electrons interact via the long range magnetostatic (curent-curent) interaction, the interaction is taken here in the mean field approximation (MFA). The tight binding Hamiltonian is of the form

$$\begin{aligned}\mathcal{H} = \sum_{nm} [&(2t + V_{nm}) c^+_{nm} c_{nm} - t e^{i\Theta_{nn+1}} c^+_{n+1m} c_{nm} + \\ &- t e^{-i\Theta_{nn+1}} c^+_{nm} c_{n+1m} - t c^+_{nm+1} c_{nm} - t c^+_{nm} c_{nm+1}],\end{aligned} \tag{5}$$

where $t = \frac{\hbar^2}{2m_e a^2}$, Θ comes from the magnetic flux ϕ

$$\Theta_{nn+1} = \frac{eaA}{\hbar c} = \frac{ea\phi}{\hbar c L_x} = \frac{e\phi}{\hbar c N}, \quad \phi = L_x A, \tag{6}$$

A is the vector potential in the x direction. The flux ϕ is composed of two parts

$$\phi = \phi_e + \phi_I, \quad \phi_I = \mathcal{L} I(\phi), \tag{7}$$

i.e. each electron moves in the external magnetic flux ϕ_e and in the flux coming from the current I in the cylinder, $\mathcal{L}$ is the selfinductance coefficient. The second term in equation (7) comes from the magnetostatic coupling.

For a clean sample ($V_{nm} = 0$) we can diagonalize the Hamiltonian directly $\mathcal{H} \equiv \mathcal{H}^{xy} = \mathcal{H}_x + \mathcal{H}_y$

$$\mathcal{H}^{xy} = \sum_{k_x} \sum_{k_y} 2t \left[1 - \cos\left(k_x a - \frac{e\phi}{\hbar c N} \right) - \cos(k_y a) \right] n_{k_x k_y}, \tag{8}$$

$$k_x = k_x(s, \phi) = \frac{2\pi}{Na} \left(s + \frac{\phi}{\phi_0} \right), \quad s = 0, \pm 1, \ldots, \tag{9}$$

$$k_y = k_y(r) = \frac{r\pi}{(M+1)a}, \quad r = 1, \ldots M, \tag{10}$$

$$\psi_{sr}(x, y) = \sqrt{\frac{2}{N(M+1)}} e^{ik_x x} \sin(k_y y), \tag{11}$$

$$\mathcal{H}^{xy} \mid sr\rangle = \mathcal{E}_{sr} \mid sr\rangle, \tag{12}$$

$$\mathcal{E}_{sr} = 2t \left[1 - \cos\left(\frac{2\pi}{N} \left(s - \frac{\phi}{\phi_0} \right) \right) - \cos\left(\frac{r\pi}{M+1} \right) \right] \equiv \mathcal{E}_s + \mathcal{E}_r. \tag{13}$$

The eigenstates of equation (5) we denote by $\mid \alpha\rangle$, these are the eigenstates of the $\mathcal{H}$ with impurities which can be diagonalized by computer calculations. Here we can treat the effects of disorder in rough analogy to the effects of temperature assuming that the elastic scattering causes transitions between the states of the perfect system [5]. We can expand the eigenstates of disordered cylinder $\mid \alpha\rangle$ in terms of those of the perfect cylinder $\mid sr\rangle$,

$$| \alpha\rangle = \sum_{sr} a_{sr}^{\alpha} \mid sr\rangle, \quad a_{sr}^{\alpha} = \langle sr \mid \alpha\rangle, \tag{14}$$

$$\mathcal{H} \mid \alpha\rangle = \mathcal{E}_{\alpha} \mid \alpha\rangle, \quad \rho_0 = \frac{1}{\exp[\beta(\mathcal{H} - \mu)] + 1}, \quad \rho_0 \mid \alpha\rangle = f_{\alpha} \mid \alpha\rangle, \tag{15}$$

where $f_{\alpha} = \frac{1}{\exp[\beta(\mathcal{E}_{\alpha} - \mu)]+1}$ is the FD distribution function, μ is the chemical potential.

In this paper we consider the case where the number of electrons N_t is kept fixed and the chemical potential depends on flux; $\mu \equiv \mu(N_t, \phi)$ can be calculated from the equation $\sum_{\alpha} f_{\alpha} = N_t$.

To introduce relaxation we assume a small coupling exists between the cylinder and the thermal bath, with the relaxation rate denoted by γ. Then following [2], by use of the kinetic equation for the density matrix $\hat{\rho}(t)$ we calculate the response of the system in the relaxation time approximation. For details of the formalism see [2].

Now we apply a small time dependent flux $\delta\phi = L_x \delta A$, where δA has only the tangential component. Denoting by $\delta\mathcal{H}$ the corresponding change in the Hamiltonian we can write

$$\delta\mathcal{H} \equiv -\frac{\delta\phi}{c} \hat{I}_i = -\frac{\delta\phi}{c} \hat{I}_p - \frac{1}{2} \left(\frac{\delta\phi}{c}\right)^2 \hat{D}, \tag{16}$$

where $\hat{I}_i = \hat{I}_p + \hat{I}_d$, $\hat{I}_i$ is the induced current operator.

$$\hat{I}_p = i \frac{e\hbar}{2m_e a L_x} \sum_{nm} (e^{i\Theta_{nn+1}} c_{n+1m}^{+} c_{nm} + h.c.). \tag{17}$$

$\hat{I}_p$ is the operator of paramagnetic current, running along the cylinder circumference, $\hat{I}_d$ is the diamagnetic current operator

$$\hat{I}_d = -\frac{\delta\phi}{c L_x^2} \hat{D}, \quad \hat{D} = \frac{e^2}{2m_e} \sum_{nm} (e^{i\Theta_{nn+1}} c_{n+1m}^{+} c_{nm} + h.c.). \tag{18}$$

Calculating by the second order perturbation theory the changes in the energy caused by $\delta\mathcal{H}$ and comparing them with the Taylor expansion of the energy we obtain important relations

$$\langle \alpha \mid \hat{I}_p \mid \alpha \rangle = -c \frac{\partial \mathcal{E}_\alpha}{\partial \phi}, \tag{19}$$

$$\frac{1}{L_x^2} \langle \alpha \mid \hat{D} \mid \alpha \rangle + 2 \sum_\beta \frac{\mid \langle \alpha \mid \hat{I}_p \mid \beta \rangle \mid^2}{\mathcal{E}_\alpha - \mathcal{E}_\beta} = c^2 \frac{\partial^2 \mathcal{E}_\alpha}{\partial \phi^2}. \tag{20}$$

Calculating the current induced by the small time dependent flux $I_i = \mathrm{Tr} \hat{I}_i \hat{\rho}$ we can find the conductivity $\sigma(w)$

$$J(w) = \frac{L_x}{V} I_i(w) = \sigma(w) E(w), \tag{21}$$

where

$$E(w) = \frac{iw}{cL_x} \, \delta\phi. \tag{22}$$

Making use of (19) and (20) we get after some algebra

$$\sigma(w) = \frac{L_x^2}{V} \left[-\frac{ic}{w} \frac{\partial I}{\partial \phi} - \frac{c^2}{\gamma - iw} \sum_\alpha \frac{\partial f_\alpha}{\partial \phi} \frac{\partial \mathcal{E}_\alpha}{\partial \phi} + i {\sum_{\alpha\beta}}' \frac{f_\alpha - f_\beta}{\mathcal{E}_\alpha - \mathcal{E}_\beta} \frac{\mid \langle \alpha \mid \hat{I}_p \mid \beta \rangle \mid^2}{\mathcal{E}_\alpha - \mathcal{E}_\beta - w - i\gamma} \right], \tag{23}$$

where I is the equilibrium persistent current

$$I = \mathrm{Tr} \rho_0 \hat{I}_p = -c \sum_\alpha f_\alpha \frac{\partial \mathcal{E}_\alpha}{\partial \phi}, \tag{24}$$

V denotes the volume of the sample.

Up to now the formal calculations went in an analogous way as in [2], the new physics appears as we perform the summation about the transverse channels. The first two terms in (23) are present only in mesoscopic multiply connected structures where the energy levels are flux sensitive. The first term, being purely imaginary, determines coherent response of the system. The second term involves the intralevel scattering and depends essentially on γ, the third term involves interlevel transitions and is connected with the elastic scattering. In the absence of the elastic scattering $\hat{I}_p$ is diagonal in the unperturbated basis $\mid sr \rangle$ and the third term is zero. The ac conductivity

in a multichannel metallic ring in the diffusive regime has been extensively discussed in [6].

In this paper we will focus our attention on the reactive response in the static limit. We want to establish the conditions under which the system exhibits strong coherent behaviour.

Calculating the $w = 0$ limit of equation (23) we find

$$\lim_{w\to 0} w\,\mathrm{Im}\,\sigma(w) = -\frac{c}{V}\frac{\partial I}{\partial \phi}. \tag{25}$$

Thus the conductivity exhibits an imaginary part which in the low frequency limit is proportional to the flux derivative of persistent current.

It follows from the basic theory of conductivity [7] that coherent electrons i.e. those which run without scattering obey the following relation

$$\lim_{w\to 0} w\,\mathrm{Im}\,\sigma(w) = \frac{n_c e^2}{m_e}, \tag{26}$$

where n_c is the density of coherent electrons. From equations (2), (3), (21) and (25) we find

$$\lim_{w\to 0}\lim_{\underline{q}\to 0} K(q,w) \equiv -\chi = -\frac{c}{V}\frac{\partial I}{\partial \phi}, \tag{27}$$

or from (26)

$$\chi = -\frac{n_c e^2}{m_e}. \tag{28}$$

The finite limit of the kernel $K(\underline{q}, w)$ corresponds to free acceleration of the part ($\equiv n_c$) of electrons - the feature characteristic of superconductors.

Using the relation between $\sigma(w)$ and $\chi(w)$ [8]

$$\sigma(w) = -\frac{i}{w}\chi(w) \tag{29}$$

we can calculate the static orbital susceptibility

$$\chi = \lim_{w\to 0}\chi(w) = \lim_{w\to 0} iw\sigma(w) = -\lim_{w\to 0} w\mathrm{Im}\sigma(w) = -\frac{n_c e^2}{m_e}, \tag{30}$$

where relation (26) has been used.

We would like to stress that the equilibrium susceptibility and the zero frequency limit of a dynamic response coincide [9] as we are working with the constant number of particles. From equations (20), (24) and (27) we obtain

$$\chi = \chi_D + \chi_p, \tag{31}$$

where

$$\chi_D = -\frac{1}{V}\sum_\alpha f_\alpha \langle \alpha \mid \hat{D} \mid \alpha \rangle = -\frac{e^2}{m_e} n, \tag{32}$$

χ_D is the diamagnetic susceptibility, n is the electron density

$$\chi_p = -\frac{L_x^2}{V}\left[2\sum_{\alpha\beta} f_\alpha \frac{\mid \langle \alpha \mid \hat{I}_p \mid \beta \rangle \mid^2}{\mathcal{E}_\alpha - \mathcal{E}_\beta} + c^2 \sum_\alpha \frac{\partial f_\alpha}{\partial \phi}\frac{\partial \mathcal{E}_\alpha}{\partial \phi}\right] \tag{33}$$

χ_p is the paramagnetic susceptibility.

Equations (30) - (32) give the following relation for χ_p

$$\chi_p = \frac{n_n e^2}{m_e}, \quad n_n = n - n_c. \tag{34}$$

We see that n_n is the density of normal electrons which undergo different kinds of scattering processes. Both terms in χ_p depend on the shape of the FS.

For a macroscopic "normal" sample the distance between energy levels is small and the occupation probability is then a smooth function of the energy. If is then justified to replace the sum in equation (24) by an integral what leads to vanishing currents and to zero limit in equation (27). Thus in macroscopic samples χ_p and χ_D nearly cancel each other and the system displays only a small residual diamagnetism.

However the mesoscopic cylinder with $L_x \sim 1\mu m$ has large energy spacings in x direction and the occupation probability near FS changes significantly over these spacings. The sum can not be then replaced by the integral and we get a finite current I which is persistent at low T because of Quantum Size Energy Gap (QSEG) [10, 11]. The paramagnetic susceptibility is drastically reduced due to the presence of the energy gaps and the system displays an anomalous diamagnetism. This problem will be further discussed in chapter 3. We see that in mesoscopic system the paramagnetic and diamagnetic

susceptibilities fail to cancel ($n \neq n_n$) and we can describe the system by the two-fluid model.

The density of coherent electrons n_c is proportional to the flux derivative of persistent current.

We shall discuss now the current I in more detail. It's magnitude depends on the strength of the phase correlations between currents of different channels.

A large phase correlation among channel currents means that the increase of the flux ϕ results in an almost simultaneous cross of the FS by the large number of channels. The most favorable situation takes place if the separation between the last occupied level and the FS from channel to channel is nearly the same. There exists then a perfect correlation among the channel currents because the M levels cross the FS simultaneously while the flux is changed by one fluxoid - we get then the largest amplitude of the total current.

Let us assume that our mesoscopic cylinder contains a small number of impurities. The average current, where the average is taken over impurity configurations, can be calculated in the linear response approach. For the M channel system one obtains [12]

$$I(\phi) \approx \exp\left(-\frac{L_x}{2\lambda}\right) \sum_{r=1}^{M} \sum_{s=0,\pm 1}^{\pm\infty} f_{sr} I_s, \tag{35}$$

where

$$I_s = \frac{e\hbar}{m_e a L_x} \sin\frac{2\pi}{N}(s-\phi^{'}) \quad s=0,\pm 1,\ldots \quad \phi^{'} = \frac{\phi}{\phi_0}, \quad \phi_0 = \frac{hc}{e}, \tag{36}$$

λ is the mean free path. For a sufficiently clean material λ can be of the order of a few microns and the impurities do not decrease the current significantly.

The current $I(\phi)$ is periodic in $\phi^{'}$ with period 1 and can be expressed as a Fourier sum [10]

$$I(\phi) = \exp\left(-\frac{L_x}{2\lambda}\right) \sum_{m=1}^{M} \sum_{l=1}^{\infty} \frac{4T}{\pi T^*} \frac{2et}{N\hbar} \frac{\exp\left(-\frac{lT}{T^*}\right)}{1-\exp\left(-\frac{2lT}{T^*}\right)} \sin\left(\frac{2\pi l \phi}{\phi_0}\right) F(m), \tag{37}$$

where λ is the mean free path, T^* is characteristic temperature set by the level spacing at the Fermi surface for an electron moving in the x direction,

$$F(m) = \sin(k_{F_x}(m)a)\cos(lNk_{F_x}(m)a), \tag{38}$$

$F(m)$ is the factor depending on a shape of the FS, k_{F_x} is calculated from the equation for the FS.

In the tight-binding approximation the shape of the FS depends on the lattice type and on the filling factor [4].

For a 2D square lattice for the half-filled band the equation for the FS reads

$$\cos(ak_{F_x}) + \cos(ak_{F_y}) = 0 \tag{39}$$

and the equation for $F(m)$ takes the form

$$F(m) = \sin(\arccos(-\cos(ak_{F_y}(m))))\cos(lN\arccos(-\cos(ak_{F_y}(m)))). \tag{40}$$

The FS is than quadratic with the diagonals along k_x, k_y axes.

For the filling factor much less than a half we can expand the cosines in the dispersion relation (13) for small k-values and we obtain

$$\mathcal{E}_{sr} = 2t\left[\frac{1}{2}\left(\frac{2\pi}{N}\left(s - \frac{\phi}{\phi_0}\right)\right)^2 - \frac{1}{2}\left(\frac{r\pi}{M+1}\right)^2 - 1\right] \equiv \mathcal{E}_s + \mathcal{E}_r. \tag{41}$$

The FS is then circular and equation (38) reads

$$F(m) = \sin\left(2\sqrt{1 - \left(\frac{ak_{F_y}(m)}{2}\right)^2}\right)\cos\left(2lN\sqrt{1 - \left(\frac{ak_{F_y}(m)}{2}\right)^2}\right). \tag{42}$$

We can also imagine a cylinder made of a set quasi 1D mesoscopic rings stacked along y axis. The FS will be then nearly flat perpendicular to k_x direction and the formula for the current is then of the form

$$I(\phi) = \exp\left(-\frac{L_x}{2\lambda}\right) M \sum_{l=1}^{\infty} \frac{4T}{\pi T^*} \frac{2et}{N\hbar} \frac{\exp\left(-\frac{lT}{T^*}\right)}{1-\exp\left(-\frac{2lT}{T^*}\right)} \sin\left(\frac{2\pi l\phi}{\phi_0}\right) \times$$
$$\times \sin(k_{F_x}a)\cos(lNk_{F_x}a). \qquad (43)$$

In order to avoid perfect nesting in all figures we present the results for slightly less than half filled bands instead of half filled ones. The Fermi Surfaces for such cases will have then rounded corners.

In Fig.1 we present persistent currents for four different shapes of 2D FS (inserted figure). We see that the amplitude of the current increases with increasing departure of the FS from circular because the phase correlation of the channel currents increases. Insert Fig.1

We have found that for a 2D squared FS, drawn by a solid line in the inserted figure, maximal interchannel phase correlations exists for $\frac{L_x}{L_y} = 2 + \frac{n}{3}$, where n is a positive integer. For other values of the ratio $\frac{L_x}{L_y}$ the current is much weaker.

The FS of the shapes as in the inserted figure are frequently met in High T_c Superconductors with 2D conduction [13, 14].

For a 3D case with P channels in the cylinder thickness the current (24) takes a form

$$I(\phi) = \exp\left(-\frac{L_x}{2\lambda}\right) \sum_{p=1}^{P} \sum_{m=1}^{M} \sum_{l=1}^{\infty} \frac{4T}{\pi T^*} \frac{2et}{N\hbar} \frac{\exp\left(-\frac{lT}{T^*}\right)}{1-\exp\left(-\frac{2lT}{T^*}\right)} \sin\left(\frac{2\pi l\phi}{\phi_0}\right) F(m,p), \qquad (44)$$

where $F(m,p)$ is a 3D factor depending on the shape of the FS.

For a 3D cubic lattice for the half-filled band the equation for the FS is of the form

$$\cos(ak_{F_x}) + \cos(ak_{F_y}) + \cos(ak_{F_z}) = 1 \qquad (45)$$

and we get the equation for $F(m,p)$

$$F(m,p) = \sin(\arccos(1-\cos(ak_{F_y}(m)) - \cos(ak_{F_z}(p)))) \times$$
$$\times \cos(lN\arccos(1-\cos(ak_{F_y}(m)) - \cos(ak_{F_z}(p)))). \qquad (46)$$

The FS is then an octahedron with the diagonals along k_x, k_y and k_z axes.

For the filling factor much less than a half we can expand the cosines in relation (45) for small k-values and we obtain

$$1-\frac{(ak_{F_x})^2}{2}+1-\frac{(ak_{F_y})^2}{2}+1-\frac{(ak_{F_z})^2}{2}=1 \tag{47}$$

and $F(m,p)$ takes a form

$$F(m,p)=\sin\left(2\sqrt{1-\left(\frac{ak_{F_y}(m)}{2}\right)^2-\left(\frac{ak_{F_z}(p)}{2}\right)^2}\right)\times$$
$$\times\cos\left(2lN\sqrt{1-\left(\frac{ak_{F_y}(m)}{2}\right)^2-\left(\frac{ak_{F_z}(p)}{2}\right)^2}\right). \tag{48}$$

The FS is then spherical.

For a body-centered tetragonal lattice and the half-filled band the equation for the FS takes a form

$$\cos\left(\frac{ak_{F_x}}{2}\right)\cos\left(\frac{ak_{F_y}}{2}\right)\cos\left(\frac{ck_{F_z}}{2}\right)=1 \tag{49}$$

and $F(m,p)$ is

$$F(m,p)=\sin\left(2\arccos\frac{1}{\cos\left(\frac{ak_{F_y}(m)}{2}\right)\cos\left(\frac{ck_{F_z}(p)}{2}\right)}\right)\times$$
$$\times\cos\left(2lN\arccos\frac{1}{\cos\left(\frac{ak_{F_y}(m)}{2}\right)\cos\left(\frac{ck_{F_z}(p)}{2}\right)}\right). \tag{50}$$

The FS is then a cuboid.

For a face-centered lattice we have the FS at half filled band

$$\cos\left(\frac{ak_{F_x}}{2}\right)\cos\left(\frac{ak_{F_y}}{2}\right)+\cos\left(\frac{ak_{F_x}}{2}\right)\cos\left(\frac{ak_{F_z}}{2}\right)+\cos\left(\frac{ak_{F_y}}{2}\right)\cos\left(\frac{ak_{F_z}}{2}\right)=1 \tag{51}$$

and we obtain $F(m,p)$ of the form

$$F(m,p) = \sin\left(2\arccos\frac{1-\cos\left(\frac{ak_{F_y}(m)}{2}\right)\cos\left(\frac{ak_{F_z}(p)}{2}\right)}{\cos\left(\frac{ak_{F_y}(m)}{2}\right)+\cos\left(\frac{ak_{F_z}(p)}{2}\right)}\right)\times$$
$$\times\cos\left(2lN\arccos\frac{1-\cos\left(\frac{ak_{F_y}(m)}{2}\right)\cos\left(\frac{ak_{F_z}(p)}{2}\right)}{\cos\left(\frac{ak_{F_y}(m)}{2}\right)+\cos\left(\frac{ak_{F_z}(p)}{2}\right)}\right). \qquad (52)$$

The FS is then a cube.

Persistent currents can be both paramagnetic and diamagnetic depending on the sample dimensions and on the Fermi energy. Paramagnetic currents has been studied in details in [3], in this paper we will discuss mainly the diamagnetic solutions.

In Fig.2 we present diamagnetic currents for different 3D lattices. Insert Fig.2

All the above considerations show that the current in a multichannel cylinder depends on the strength of the interchannel correlations and therefore on the shape of the FS.

In the case of the spherical (circular) FS the channel currents add almost without phase correlation and the total current is small, whereas for the FS being a cuboid (square) with rounded corners the correlation is almost perfect and the total current is large.

It follows from (27) and (28) that persistent currents are carried by coherent electrons. Thus the shape of the FS determines the density of coherent electrons n_c in the sample and it increases with increasing the curvature of the FS.

3 Self-sustaining currents

Finally we discuss the possibility of spontaneous self-sustaining currents i.e. those which flow without any external field.

The magnetic flux which drives the persistent current given by equation (37) is the sum of the externally applied flux and the flux from the currents

itself. The inductance coefficient $\mathcal{L}$ depends on the sample geometry and can be large for a cylinder geometry.

Equations (7) and (37) form two selfconsistent equations for the current and it raises the possibility of a spontaneous self-sustaining current at the external flux zero. The possibility of the finite spontaneous current depends again crucially on the shape of the FS.

The phenomenon of the self-sustaining currents is a collective effect and it requires many electrons to support the current at $\phi_e = 0$. Therefore for a 2D cylinder with $d \sim 1\mathring{A}$ we do not find any spontaneous current solutions. To get it we have to consider the cylinder made of a set of 2D concentric cylindrical sheets or a 3D cylinder (in both cases we keep $d \ll R$).

The graphical solutions of the self-cosistent equations (7) and (37) at $\phi_e = 0$ are presented in figs. 3, 4 for different shapes of the FS. The intersections of the two curves marked by circles give the values of self-sustaining currents. The temperature at which the transition to the state with such currents occurs is denoted by T_c.

For diamagnetic currents presented in Fig.3 and Fig.4 the self-sustaining currents correspond to flux trapping in the cylinder - the phenomenon known in superconductivity. Insert Fig.3 Insert Fig.4

It follows from our considerations that the self-sustaining solutions are obtained only in samples where the FS has flat regions. For spherical and nearly spherical FS we do not obtain it because the density of coherent electrons is too small.

There are two reasons for the presence of coherent electrons in mesoscopic samples. At first the QSEG hampers the scattering at low temperatures. At second as will be shown below, the selfconsistent flux ϕ_I increases coherence in the sample.

To be mostly transparent let us consider a cylinder made of a set of quasi-one dimensional mesoscopic rings stacked along certain axis for the filling factor much less than 1. The energy spectrum (cf.(41)) of a single ring is given by the formula

$$\mathcal{E}_s = \frac{2\hbar^2\pi^2}{m_e L_x^2}(s - \phi^{'}) \quad s = 0, \pm 1, \ldots \quad \phi^{'} = \phi_e^{'} + \frac{\mathcal{L}I}{\phi_0}. \tag{53}$$

Assuming that each ring posses an odd number of electrons we can calculate

the energy gap at the FS, $\Delta \equiv \mathcal{E}_{s_F+1} - \mathcal{E}_{s_F}$. We find

$$\Delta = \Delta_0 \left(1 - 2\phi_e^{'} + 2\frac{\mathcal{L} \mid I \mid}{\phi_0}\right), \tag{54}$$

where $\Delta_0 = \frac{h^2}{2m_e L_x^2} N$ is the QSEG.

We see that Δ contains a term Δ_d:

$$\Delta_d \equiv \Delta_0 \frac{\mathcal{L} \mid I \mid}{\phi_0}, \tag{55}$$

coming from the magnetostatic interactions among electrons - Δ_d is the dynamic part of the energy gap, which has to be calculated in a self-consistent way.

If we calculate the energy of electrons in the cylinder made of a set of M rings for $\phi^{'} < \frac{1}{2}$ we find

$$E(\phi^{'}) = M \frac{2\hbar^2 \pi^2}{m_e L_x^2} \sum_{s=0,\pm 1}^{\pm s_F} (s - \phi^{'})^2 < E(\phi_e^{'}), \tag{56}$$

because $\phi^{'} = \phi_e^{'} + \frac{\mathcal{L} I}{\phi_0} < \phi_e^{'}$ as the current is diamagnetic for $\phi^{'} < \frac{1}{2}$.

The gain in the energy in equation (56) may be called the condensation energy due to orbital magnetic interactions.

In case of an even number of electrons in each ring, spontaneous flux ϕ_{sp} is created in a system [11]. ϕ_{sp} shifts the energy levels and as a result a gap appears $\Delta = \mathcal{E}_{-s_F} - \mathcal{E}_{s_F}$.

One finds

$$\Delta = \Delta_d = \Delta_0 \cdot 2\phi_{sp}^{'}, \quad \phi_{sp} = \mathcal{L} I_{sp}, \tag{57}$$

I_{sp} is the spontaneous current. This gap is dynamic, because it results from the collective action of all electrons which produces spontaneous flux in order to minimize the energy.

In both cases the flux coming from the currents produces a dynamic gap and therefore increases coherence of electrons. This mechanism of gaining the energy is valid only in mesoscopic systems as $\Delta_d \to 0$ for $\Delta_0 \to 0$. Similar considerations can be performed for electrons moving on a cylinder with the conclusion that the orbital magnetic interactions (taken here in the MFA) increase coherence.

It can easily be seen from equation (33) that increase of an energy gap results in a further depression of χ_p because it decreases both terms in the formula for paramagnetic susceptibility thus enhancing the coherent response of the system.

This phenomenon is analogous to the reduction of the paramagnetic susceptibility in superconductors and in organic molecules due to pair correlation [15].

4 Conclusions

It is already well established both theoretically [10] and experimentally [16] that persistent currents which "never decay" can flow in mesoscopic metallic or semiconducting rings being a manifestation of quantum coherence. Such currents were previously attributed solely to superconductors.

The question arises what are the similarities and differences between the properties of mesoscopic systems and superconductors [11, 17]. Superconductivity is a collective phenomenon and follows from an attractive interaction. To compare the two phenomena we considered electrons interacting via the orbital magnetic long range interaction and moving on a thin-walled hollow cylinder. The interaction, taken here in the MFA, means that each electron, besides the external flux ϕ_e feels a magnetic flux ϕ_I coming from the currents. The selfinductance of the one-dimensional loop is negligible [2], however, it can be substantial for a system of cylindrical geometry.

We calculated the frequency dependent conductivity and the electromagnetic kernel. The conductivity is strongly related to the presence of persistent currents. We have shown that

$$\lim_{w\to 0}\lim_{\underline{q}\to 0} K(\underline{q}, w) = -c\frac{\partial I}{\partial \phi} \tag{58}$$

The finite limit of the kernel is equivalent to the infinite conductivity. It is absent in macroscopic normal metals and present in superconductors. Thus although both the elastic and inelastic scattering was taken into account, part of electrons ($\equiv n_c$) is in a coherent state and moves without dissipation. This feature is connected with the multiply connected structure of our cylinder where a change of phase produced by the magnetic flux can not be removed

by a gauge transformation and results in an equilibrium current I which is persistent at low temperatures.

In a one-dimensional mesoscopic ring the current I is simply a diamagnetic or paramagnetic reaction to the external flux ϕ_e and must vanish at $\phi_e = 0$ [2]. The situation looks different in a multichannel cylinder. We have shown that the self-sustaining, persistent currents can exist at $\phi_e = 0$ in relatively clean samples (ballistic regime) and for the FS which are sufficiently flat. The self-sustaining solutions correspond to "orbital ferromagnetism" for paramagnetic currents (this case was discussed in [3]) and to flux trapping for diamagnetic currents - a feature characteristic of superconductivity.

The coherent response and related quantities depend on the sample dimensions and on the geometry of the FS. We have shown that the magnitude of a persistent current and of a dynamic and static response function depend on the phase correlation of currents from different channels. This correlation increases with increasing the curvature of the FS. The Fermi Surfaces with flat regions follow in a natural way in the tight-binding model.

The coherent response of a mesoscopic hollow cylinder follows from two reasons. At first because of finite size the energy spectrum is discrete and QSEG causes that the diamagnetic and paramagnetic parts of susceptibility fail to cancel - this is connected with the single electron properties. At second the orbital magnetic interaction among electrons leads to appearance of the dynamic gap and it further increases coherence. It manifests e.g. in further decrease of the paramagnetic susceptibility and can lead to the coherent collective phenomena such as "orbital ferromagnetism" or flux trapping.

It follows from our considerations that a mesoscopic cylinder can be in general described by the two-fluid model. The coherent behaviour is determined here by the interplay between finite size effects and the correlations coming from the orbital magnetic interaction. The energy levels are periodic functions of the flux with period ϕ_0 and the minima of the total energy determine stable values of flux contained in the cylinder. We will discuss it at related problems in more detail in a subsequent paper.

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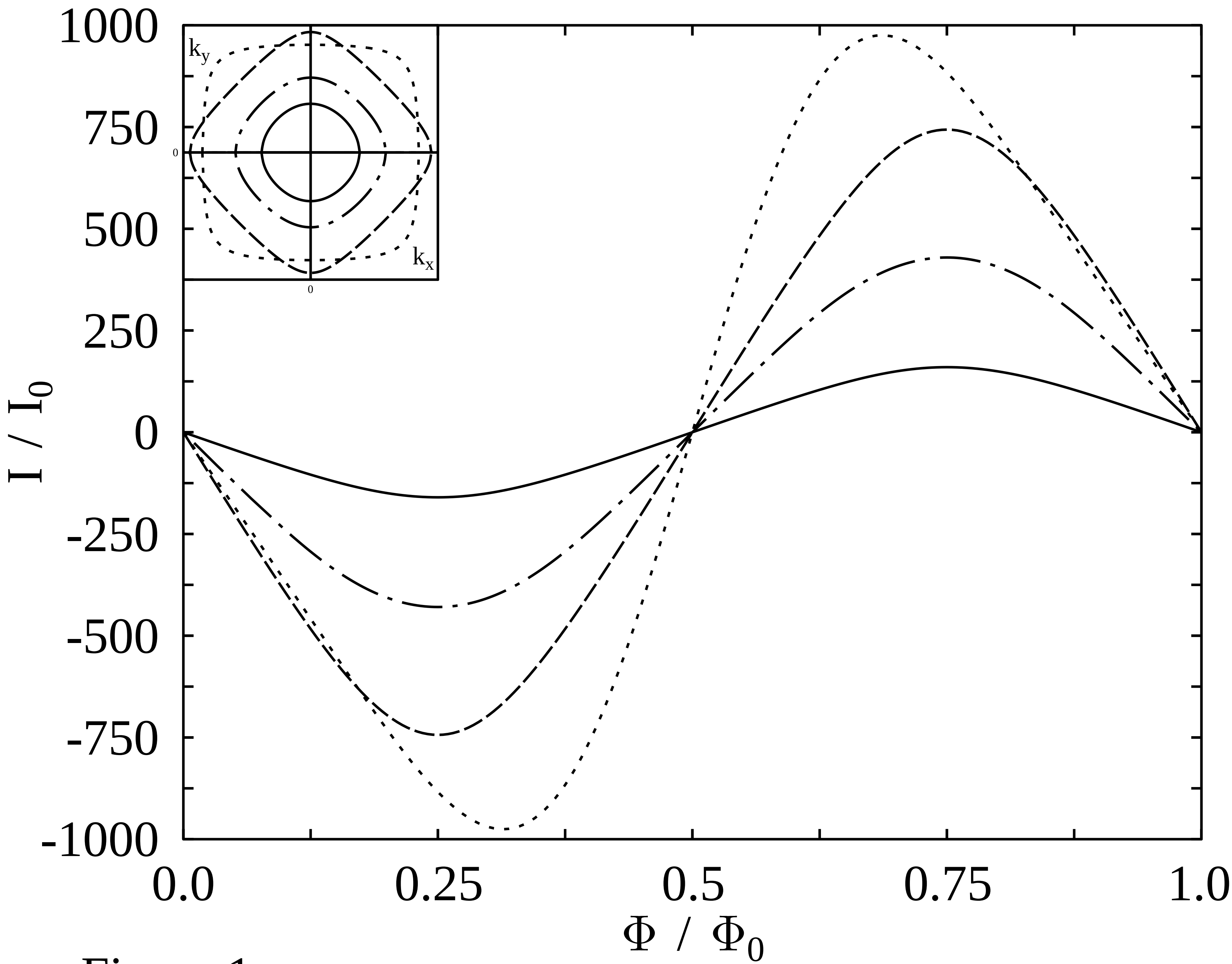
Figure captions

Figure 1. Persistent currents in I_0 units (where $I_0 = \frac{2et}{N\hbar}\sin(k_F a)$) vs flux for different shapes of the 2D Fermi surfaces for $T = 1.21K, E_F = 7eV$ and $\frac{L_x}{L_y} = 3$. In the inserted figure the shapes of the Fermi surfaces are shown.

Figure 2. Diamagnetic persistent currents vs flux for different 3D lattices and slightly less than half filled band: cubic (solid line), body-centered tetragonal (dotted line), face-centered (dashed line) and cubic for filling factor much less than 1 (dash-dotted line) for $T = 1.21K, E_F = 7$ eV and $L_x = 3.14\mu m, L_y = 0.98\mu m, L_z = 0.01\mu m$.

Figure 3. The graphical solution of a set of self-consistent eqs. (37) and (7) for different temperatures and for body-centered tetragonal lattice for $E_F = 8$ eV and $L_x = 3.14\mu m, L_y = 0.94\mu m, L_z = 0.02\mu m$. The nonzero crossings of the straight line (7) with the current-flux characteristic (37) denoted by circles correspond to flux trapped in mesoscopic cylinder.

Figure 4. The graphical solution of a set of self-consistent eqs. (37) and (7) for different temperatures and for face-centered lattice. for $E_F = 6$ eV and $L_x = 3.14\mu m, L_y = 0.96\mu m, L_z = 0.03\mu m$. The nonzero crossings of the straight line (7) with the current-flux characteristic (37) denoted by circles correspond to flux trapped in mesoscopic cylinder.



1000
750
500
250
0
-250
-500
-750
-1000
$I \;/\; I_0$
0.0
0.25
0.5
0.75
1.0
$\Phi \;/\; \Phi_0$
k_y
k_x
0
0

Figure 1

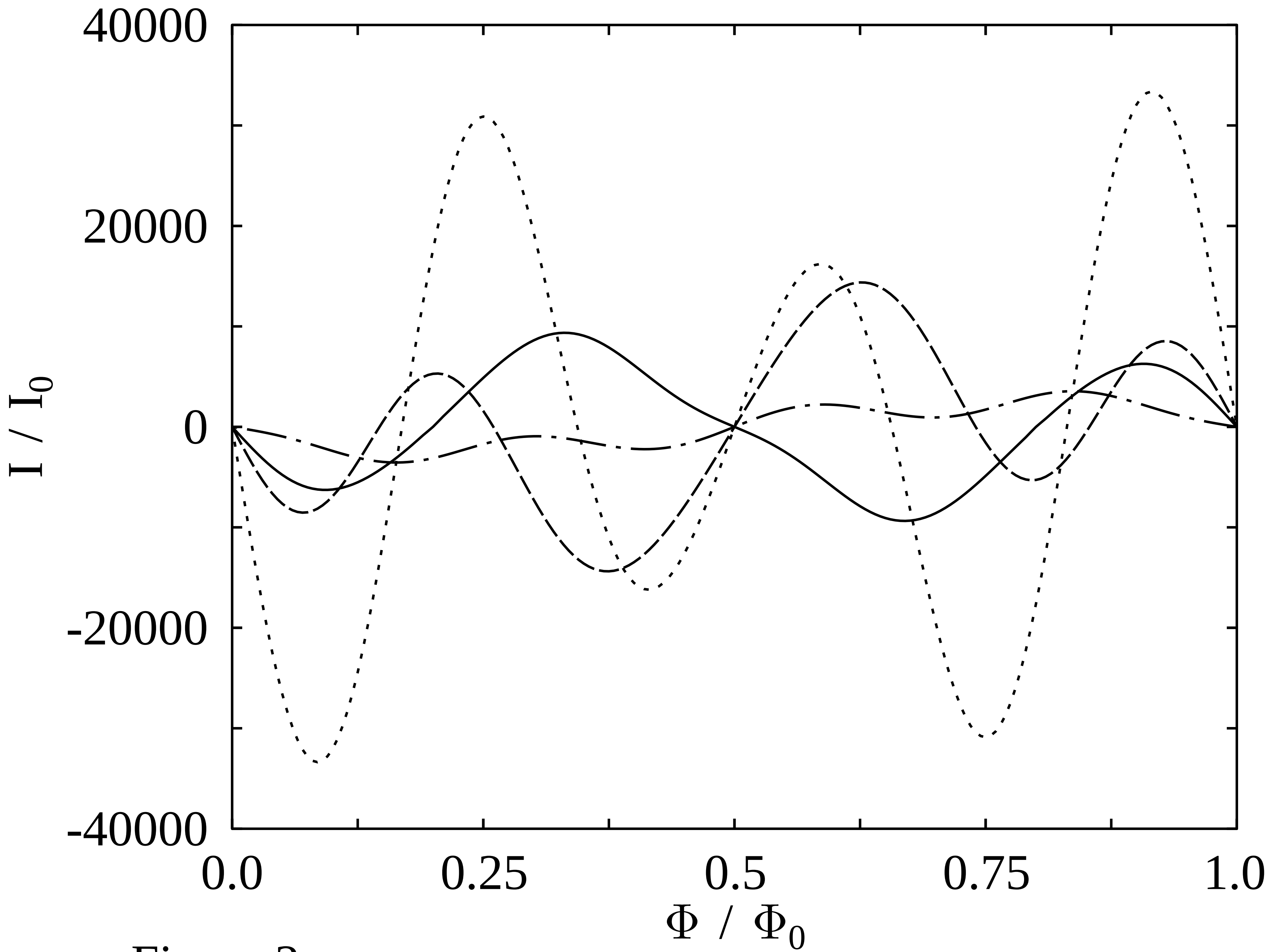
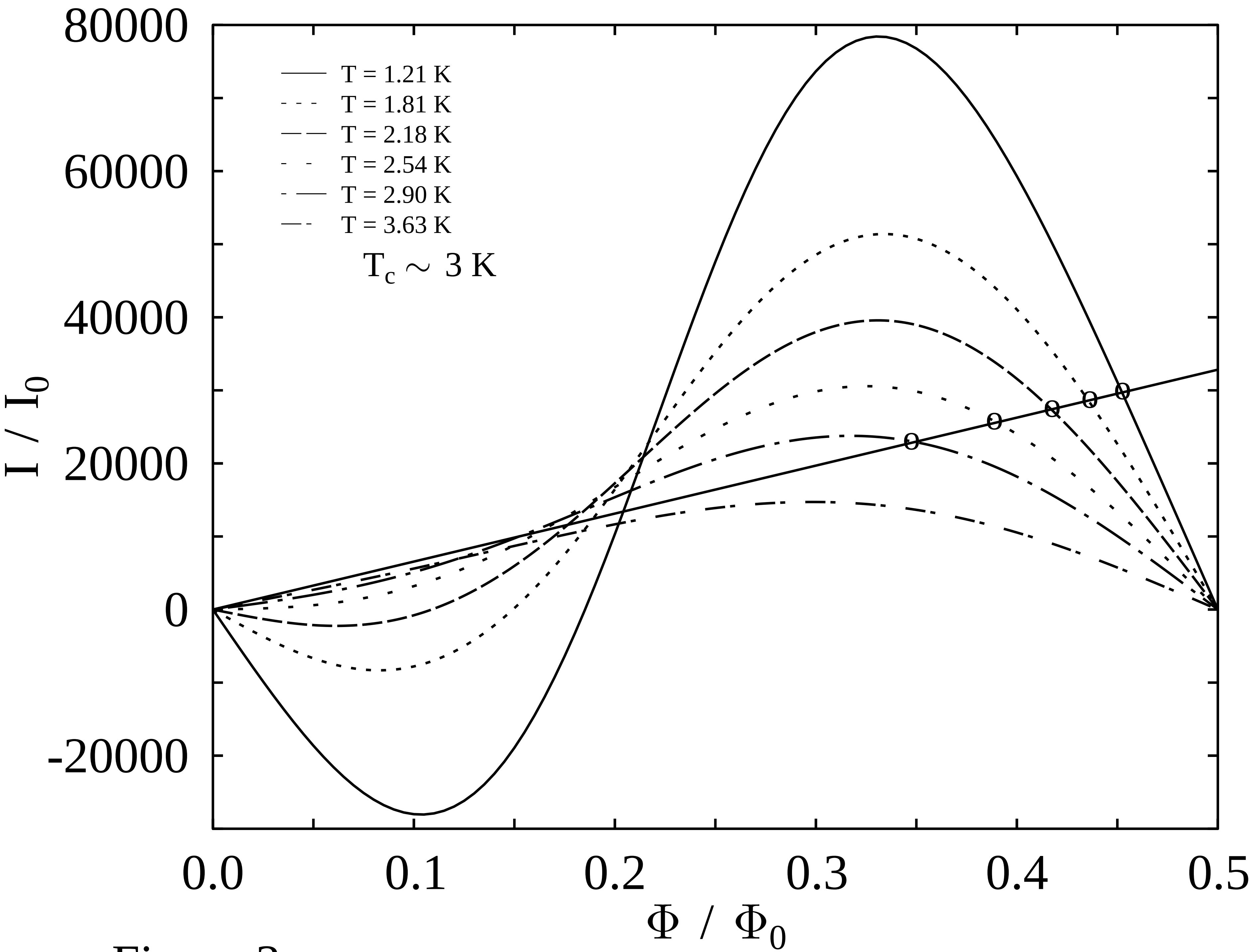
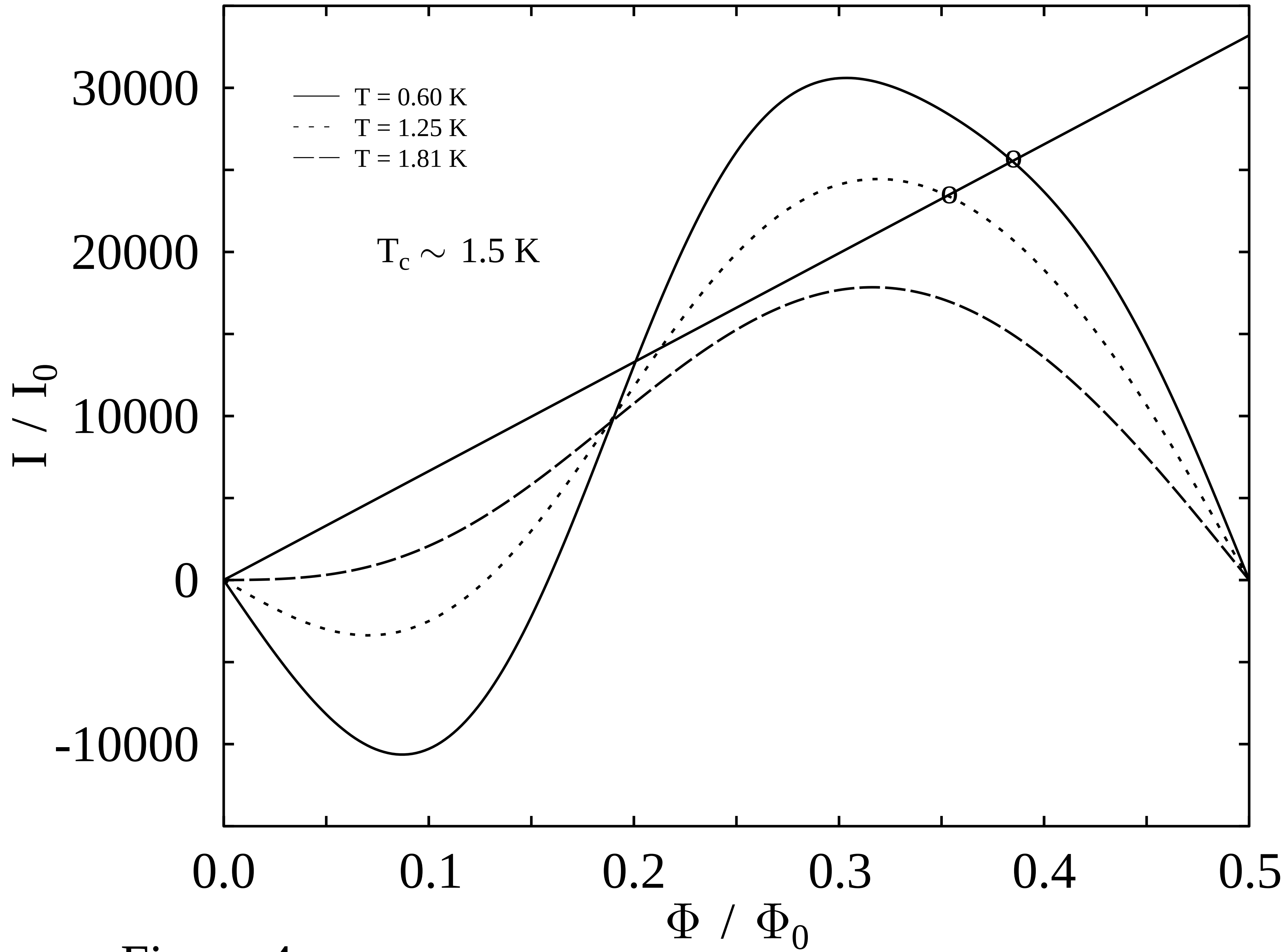


Figure 2



80000
60000
40000
20000
0
-20000
I / I0
0.0
0.1
0.2
0.3
0.4
0.5
Φ / Φ0
T = 1.21 K
T = 1.81 K
T = 2.18 K
T = 2.54 K
T = 2.90 K
T = 3.63 K
Tc ~ 3 K

Figure 3



T = 0.60 K
T = 1.25 K
T = 1.81 K
T$_c$ ~ 1.5 K
30000
20000
10000
0
-10000
I / I$_0$
0.0
0.1
0.2
0.3
0.4
0.5
Φ / Φ$_0$

Figure 4